

Analytic Combinatorics, ECCO 2012

Exercise sheet 2

- Find the ordinary generating function for the following combinatorial classes:
 - Compositions of n .
 - Compositions of n with exactly k parts.
 - Compositions of n where each part has size at most r .
 - Partitions of n .
 - Partitions of n with at most k parts.
 - Partitions of n where each part has size at most r .
- Non-plane trees. An *unordered tree*, also called *non-plane tree*, is just a tree in the general graph-theoretic sense, so that there is no order between subtrees emanating from a common node. Find the generating function for the set of all rooted non-planar trees on n vertices.
- Let $W^{(k)}$ be the collection of words in an alphabet $\{a, b\}$ which do not have k consecutive a 's. Find its ordinary generating function.
 - What is the OGF for an alphabet of cardinality m ?
- Cyclic compositions. The class $D = CYC(\mathcal{I})$ comprises compositions defined up to circular shift of the summands; so, for instance $2 + 3 + 1 + 2 + 5$, $3 + 1 + 2 + 5 + 2$, etc, are identified. Find the generating function $D(z)$ for fixed k , and verify that the number of cyclic partitions of n of size k is

$$D_n = \frac{1}{n} \sum_{k|n} \varphi(k)(2^{n/k} - 1),$$

where the condition " $k | n$ " indicates a sum over the integers k dividing n .