

## Expected Value & Variance

<i>Distribution</i>	<i>Description</i>	<i>Prob. Mass function</i>	$E[X]$	$\text{Var}[X]$
<b>Uniform</b>	$X$ takes each value in the range $R$ with equal probability. $R$ contains $n$ numbers. <i>Example:</i> Value shown on a fair die.	$f_X(k) = \begin{cases} \frac{1}{n} & k \in R \\ 0 & \text{else.} \end{cases}$	$\frac{1}{n} \sum_{k \in R} k$ (average)	$\frac{1}{n} \sum_{k \in R} (k - E[X])^2$
<b>Bernoulli</b>	A Bernoulli trial results in “success” with probability $p$ and “failure” with probability $1 - p$ . $X$ takes the value 1 in the event of success, and 0 for failure.	$f_X(k) = \begin{cases} p & k = 1 \\ 1 - p & k = 0 \\ 0 & \text{else.} \end{cases}$	$p$	$p(1 - p)$
<b>Binomial</b>	A Bernoulli trial with probability $p$ of success is performed $n$ times. $X$ is the number of successes.	$f_X(k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & 0 \leq k \leq n \\ 0 & \text{integer} \\ 0 & \text{else.} \end{cases}$	$np$	$np(1 - p)$
<b>Geometric</b>	A Bernoulli trial with probability $p$ of success is performed until the first success is achieved. $X$ is the number of failures before the first success.	$f_X(k) = \begin{cases} p(1 - p)^k & k \geq 0 \\ 0 & \text{integer} \\ 0 & \text{else.} \end{cases}$	$\frac{1 - p}{p}$	$\frac{1 - p}{p^2}$
<b>Hypergeometric</b>	A jar contains $N$ marbles, of which $m$ are white and $N - m$ are black. A sample of $n$ marbles is drawn, and $X$ is the number of white marbles in the sample.	$f_X(k) = \begin{cases} \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}} & 0 \leq k \leq m \\ 0 & \text{integer} \\ 0 & \text{else.} \end{cases}$	$\frac{mn}{N}$	$\frac{nm(N - m)(N - n)}{N^2(N - 1)}$ (not in slides)
<b>Poisson</b>	$X$ is the number of times an event occurs, which is known to occur at an “average rate” $\lambda$ , independently of the amount of time since the last event.	$f_X(k) = \begin{cases} \frac{e^{-\lambda} \lambda^k}{k!} & k \geq 0 \\ 0 & \text{integer} \\ 0 & \text{else.} \end{cases}$	$\lambda$	$\lambda$

1. PRACTICE PROBLEMS

- (1) Derive the formula for  $E[X]$  for the Bernoulli random variable.
- (2) Show that  $E[X]$  for the Binomial and Hypergeometric random variables can be derived from the previous answer using the properties of Expected Value.
- (3) Why is  $\text{Var}[X]$  for the hypergeometric random variable not given by  $\frac{mn(N-m)}{N^2}$ ?
- (4) What is a fair price for the following games:
  - (a) A fair coin is flipped until heads comes up, and the player wins a dollar for each tails she flipped before the heads.
  - (b) The game runner and the player each roll a six-sided die. If the player has a strictly higher roll, then he wins a dollar.
  - (c) The player watches a road which has red cars pass at a rate of 10 per hour. She wins a dollar for each red car she sees in the hour.
- (5) Suppose I flip a penny, a nickel, a dime, and a quarter. Let  $X$  be the value in cents of the coins showing heads. What is  $E[X]$  and  $\text{Var}[X]$ ?