

(Problems taken from worksheet by Rob Bayer)

Trig Substitution

(1) Consider the integral $\int \frac{x}{\sqrt{9-x^2}} dx$.

- Find this integral using an appropriate trig substitution.
- Find this integral using an ordinary u-substitution.
- Are your answers the same?

(2) Calculate:

- $\int \frac{dx}{x^2\sqrt{16-x^2}}$.
- $\int \frac{\cos t}{\sqrt{1+\sin^2 t}} dt$.

(3) True/False. For those that are true, prove it. For those that are false, give a counterexample.

- $\sin^{-1}(\sin x) = x$ for all x .
- $\sqrt{\sec^2(x) - 1} = \tan x$.
- $\sin(\sin^{-1} x) = x$ for all x in the domain of \sin^{-1} .

Partial Fractions

(1) Give the form of the partial fraction decomposition for each of the following. You need not solve for A, B, C , etc.

- $\frac{x^3 + 2x - 1}{x^4 - 3x^3 - 4x^2}$.
- $\frac{3x^2 - 7x}{(x-2)^2(x^2+x+1)^2x^3}$.
- $\frac{x^3 - 1}{x^2 - x}$.

(2) Consider the integral $\int \frac{dx}{x^2 - 4}$.

- Find this integral using partial fractions.
- Find this integral using trig substitution.
- Are your answers the same?

(3) Calculate the following:

- $\int \frac{6x^3 + 7x^2 - 2x - 5}{x^4 - x^2} dx$.
- $\int \frac{3e^{2t}}{e^{2t} - e^t - 6} dt$.