

## Integration Review

$$(1) \int \sin x - \cos x + 1 \, dx.$$

$$= -\cos x - \sin x + x + C.$$

$$(2) \int x + \frac{1}{x} \, dx.$$

$$= \frac{x^2}{2} + \ln|x| + C.$$

$$(3) \int_1^8 x \sqrt{x-1} \, dx.$$

Let  $u = x-1$ . Then,  $du = dx$ ,  $x = u+1$

$$\begin{aligned} &= \int_0^7 (u+1) u^{1/3} \, du = \int_0^7 u^{4/3} + u^{1/3} \, du = \left. \frac{3}{7} u^{7/3} + \frac{3}{4} u^{4/3} \right|_0^7 \\ &= \frac{3}{7} \cdot 7^{7/3} + \frac{3}{4} \cdot 7^{4/3} = 21 \sqrt[3]{7} + \frac{21}{4} \sqrt[3]{7} = \frac{105 \sqrt[3]{7}}{4} \end{aligned}$$

$$(4) \int_0^2 \frac{5x}{x^2+5} \, dx.$$

Let  $u = x^2+5$ .  $du = 2x$ .

$$= \int_5^9 \frac{5}{2} \frac{du}{u} = \frac{5}{2} \ln u \Big|_5^9 = \frac{5}{2} (\ln 9 - \ln 5)$$

## Integration by Parts

(1)  $\int t \sin(t) dt.$

$$\left[ \begin{array}{ll} u = t & dv = \sin t \, dt \\ du = dt & v = -\cos t \end{array} \right]$$

$$\Rightarrow \int t \sin t \, dt = -t \cos t + \int \cos t \, dt$$

$$= \boxed{-t \cos t + \sin t + C.}$$

$$(2) \int_4^9 \frac{\ln y}{\sqrt{y}} dy. = 2y^{1/2} \ln y - \int \frac{2y^{1/2}}{y} dy \quad \left[ \begin{array}{ll} u = \ln y & dv = y^{-1/2} dy \\ du = \frac{dy}{y} & v = 2y^{1/2} \end{array} \right]$$

$$= 2y^{1/2} \ln y - \int 2y^{-1/2} dy$$

$$= 2y^{1/2} \ln y - 4y^{1/2} \Big|_4^9 = (6 \ln 9 - 12) - (4 \ln 4 - 8)$$

$$= \boxed{6 \ln 9 - 4 \ln 4 - 4.}$$

(3)  $\int e^{2x} \sin(x) dx.$

$$= \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$$

$$\left[ \begin{array}{ll} u = \sin x & dv = e^{2x} dx \\ du = \cos x \, dx & v = \frac{1}{2} e^{2x} \end{array} \right]$$

$$\int e^{2x} \cos x \, dx =$$

$$\frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x \, dx$$

$$\left[ \begin{array}{ll} u = \cos x & dv = e^{2x} dx \\ du = -\sin x \, dx & v = \frac{1}{2} e^{2x} dx \end{array} \right]$$

$$\Rightarrow I = \frac{1}{2} e^{2x} \sin x - \frac{1}{2} \left( \frac{1}{2} e^{2x} \cos x + \frac{1}{2} I \right) \Rightarrow \frac{5}{4} I = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x$$

(4)  $\int \tan^{-1}(x) dx.$

$$\Rightarrow I = \boxed{\int e^{2x} \sin x \, dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + C.}$$

$$= x \tan^{-1}(x) - \int \frac{x}{1+x^2} dx$$

$$\int \frac{x}{1+x^2} dx$$

$$\left[ \begin{array}{l} \text{Let } w = 1+x^2 \\ dw = 2x \, dx \end{array} \right]$$

$$\left[ \begin{array}{ll} u = \tan^{-1}(x) & dv = dx \\ du = \frac{dx}{1+x^2} & v = x \end{array} \right]$$

$$= \frac{1}{2} \int \frac{dw}{w} = \frac{1}{2} \ln |1+x^2| + C$$

$$\text{So, } \int \tan^{-1}(x) dx = x \tan^{-1}(x) - \frac{1}{2} \ln |1+x^2| + C.$$