Quiz 6

MATH 1B, SPRING 2012

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SECTION: NAME: SOLUTION

Determine whether the following series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{n+5}.$$

i) Absolute Convergence, i.e. convergence of

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+5}.$$

We use the Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.

$$\lim_{n\to\infty} \frac{|a_n|}{b_n} = \lim_{n\to\infty} \frac{n}{n+5} = 1, \text{ a finite limit } > 0.$$

Since $\frac{1}{2} < 1$, $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges $\Rightarrow \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+5}$ diverges.

ii) Conditional Convergence; convergence of $\sum_{n=1}^{\infty} (-1)^n b_n$

We use the Alternating Series Test. $b_n = \frac{\sqrt{n}}{n+5}$

1. $b_n > 0$ ✓

2. $\lim_{n\to\infty} \frac{\sqrt{n}}{n+5} = \lim_{n\to\infty} \frac{1}{\sqrt{n} + \frac{5}{\sqrt{n}}} = 0.$ ✓

3. $b_{n+1} \leq b_n$. Let $f(x) = \frac{\sqrt{x}}{x+5}$. $f'(x) = \frac{(x+5)\frac{1}{2\sqrt{x}} - \sqrt{x}}{(x+5)^2}$

$$= \frac{x+5-2x}{2\sqrt{x}(x+5)^2} = \frac{5-x}{2\sqrt{x}(x+5)^2}$$

This calculation implies that $f'(x) \leq 0$ for all $x \geq 5$. So, for $n \geq 5$, the terms of the series are decreasing, and we can ignore finitely many terms.

Therefore, our series is convergent, but not absolutely convergent.

$\Rightarrow$ Our series is conditionally convergent.