

Quiz 2

MATH 1B, SPRING 2012

February 3, 2012

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SECTION:

NAME: SOLUTION

Solve the following integral, showing all steps clearly:

$$\int_0^1 \frac{x \, dx}{x^2 + 4x + 13}$$

$$b^2 - 4ac = 16 - 4 \cdot 13 < 0 \Rightarrow \text{So, } x^2 + 4x + 13 \text{ is irreducible.}$$

For a u-substitution $u = x^2 + 4x + 13$

we need $du = (2x + 4) \, dx$.

So, we take: $x = \frac{1}{2}(2x + 4) - 2$.

$$\Rightarrow \int_0^1 \frac{x \, dx}{x^2 + 4x + 13} = \int_0^1 \frac{\frac{1}{2}(2x + 4) \, dx}{x^2 + 4x + 13} - \int_0^1 \frac{2 \, dx}{x^2 + 4x + 13}$$

① ②

For ①, we make the above u-sub: $= \frac{1}{2} \int_{13}^{18} \frac{du}{u}$
 $= \frac{1}{2} (\ln 18 - \ln 13)$

For ②, we complete the square and do a trig sub:

$$\int_0^1 \frac{2 \, dx}{x^2 + 4x + 13} = 2 \int_0^1 \frac{dx}{(x+2)^2 + 9} \left\{ \begin{aligned} &= \frac{2}{3} \tan^{-1} \left(\frac{x+2}{3} \right) \Big|_0^1 \text{ (by the formula).} \\ &= \frac{2}{3} \tan^{-1}(1) - \frac{2}{3} \tan^{-1} \left(\frac{2}{3} \right) = \frac{\pi}{6} - \frac{2}{3} \tan^{-1} \left(\frac{2}{3} \right) \end{aligned} \right.$$

Or by trig sub, $x+2 = 3 \tan \theta$. when $x=1$, $\theta = \frac{\pi}{4}$
 $dx = 3 \sec^2 \theta \, d\theta$ $x=0$ $\theta = \tan^{-1}(\frac{2}{3})$

$$= 2 \int_{\tan^{-1}(\frac{2}{3})}^{\pi/4} \frac{3 \sec^2 \theta \, d\theta}{9 \tan^2 \theta + 9} = 2 \int_{\tan^{-1}(\frac{2}{3})}^{\pi/4} \frac{1}{3} \, d\theta = \frac{\pi}{6} - \frac{2}{3} \tan^{-1} \left(\frac{2}{3} \right).$$

$$\text{So, } \int \frac{x \, dx}{x^2 + 4x + 13} = \textcircled{1} - \textcircled{2} = \frac{1}{2} (\ln 18 - \ln 13) + \frac{2}{3} \tan^{-1}\left(\frac{2}{3}\right) - \frac{\pi}{6}.$$