GSI: Zvi Rosen **Math 1B: Midterm 2 Review Questions** March 16, 2012

1. Straightforward Problems

(1) Consider the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. Perform the ratio test for convergence – is it conclusive?

(2) The series $\sum_{n=1}^{\infty} \frac{1-n}{1+n}$:

A. converges absolutely. B. converges conditionally. C. diverges.

(3) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$:

A. converges absolutely. B. converges conditionally. C. diverges.

(4) Find the binomial series for $y = (1 - \frac{x}{2})^{-1/2}$.

(5) Find the limit of the sequence $\{a_n\} = \left\{ \frac{n \ln n}{n^2 + 5} \right\}$.

(6) Which of the following series converge (state which tests/rules you use):

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\sum_{n=1}^{\infty} 2^n$ III. $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ IV. $\sum_{n=1}^{\infty} (\frac{1}{2})^n$

2. Tricky Problems

(1) Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n (1+n)}{2n} (x-5)^n.$$

Investigate the convergence on the endpoints of the interval.

(2) Do the same for:

$$\sum_{n=2}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n}}.$$

(3) Write the second-degree Taylor polynomial (T_2) for $f(x) = \sqrt{x}$ centered at $a = 100$. Use this to estimate $\sqrt{101}$. Estimate the error (R_2).

(4) Find the first four nonzero terms of the MacLaurin series for $\int_0^x \sqrt{1+t^3} dt$.

(5) Find the value of

$$\lim_{x \to 0^+} \frac{\sin x - x}{2x^3}$$

(6) What is the limit as $n \to \infty$ of the sequence

$$\left\{ \left(1 + \frac{1}{n^2}\right)^n \right\}$$

(7) Express these series as closed form functions:

I. $\sum_{k=0}^{\infty} \frac{x^{k+3}}{3^k k!}$ II. $\sum_{n=0}^{\infty} \frac{n}{n+1} x^n$ III. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2 \cdot 4 \cdot 6 \cdots (2n)}$

3. Challenge Problems

(1) Find the coefficient of x^3 in the MacLaurin series for $xe^x \cos(x/2)$.

(2) Does the series $\sum_{n=2}^{\infty} \dfrac{\log_n(n!)}{n^3}$ converge or diverge? Explain.

(3) Find the sum of the series

$$\sum_{n=3}^{\infty} \ln[(\frac{n}{n+1})^3]$$

(4) Find $f^{(5)}(3)$ where $f(x) = x\ln(x) - 3\ln(x)$.

(5) (a) Find the Taylor Series of $\dfrac{1}{1-x}$ centered at $a = -2$. Find the radius of convergence.
(b) Based on this calculation, evaluate:

$$\sum_{n=0}^{\infty} \frac{-(3-e)^n}{n \cdot 3^n}$$